Output Feedback Controller of Electric Drive based Variable Speed Induction Motors

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Abstract:
The main purpose of this study is design of output feedback controller based sampled – output full order high gain observer and its application on variable speed nonlinear model of induction machine drives. The proposed new state observer over standard ones is robust with respect to sampling time schedule and unknown external load torque profile to the observer. Only two of the electrical state variables are supposed to be accessible, which are stator voltages and discrete time mode stator currents. As a matter of fact, this study presents an observability analysis of the induction machine, leading to an adequate observability condition. Nonlinear output feedback control design technique ensures fast tracking time response and exponential convergence of the observation error with time progressive. The stability convergence of the proposed state observer is formally analyzed using tools of Lyapunov stability theory. The simulation results using MATLAB environments are given to support the main results and ensure the efficiency of the proposed strategy applied on 7.5 kW AC machine based on time varying gain inside the observer structure. It plays an active role in increasing the sampling time interval compared to fixed gain starting from 1.5ms upto 4ms

Keywords:- State observer; Induction machine; Lyapunov theory; Sampled output; Output feedback controller.

1. INTRODUCTION
It is worthy emphasized that the induction motor is one of the main actuators for domestic and industrial purposes. In fact, as compared to the DC motor, Induction machine has a better power/mass ratio, simpler maintenance and a relatively lower cost. Motivated by the fact that the controlling and observation of the induction motor is very complex and this complexity resides in numerous machine state variables, strong nonlinearity nature for this type of alternating machines [14]. Also, some of the machine state variables are inaccessible. If some of those variables are accessible, but their measurements are impractical [4]. The magnetic state variable related to flux measurement is inaccessible in many of the domestic purposes and industrial process.
However, many of the studies considered the rotor speed accessible. Other studies had been used different approaches for synthesis of state observer coupled with three – phase induction machine drives using Luenberger state observer for speed observation [11], the linear matrix inequality approach [5], sliding mode observer for sensorless control of induction motors [3]. This allows many of research activities concerning with the synthesis of nonlinear observer for a class of variable speed induction drives based on input/output injective measurements in continuous time mode as claimed in [9]. Subsequently, many of scientists are faced the problem of state estimating and intensive research activities are addressed on this topic [7]. Unfortunately, most of the proposed observer design techniques offer continuous time state observations that need discretization for practical implementation and realization of control laws. Also, exact discretization is a highly complex issue due to the strong nonlinearity nature of the state observer and there is no guarantee that approximate discrete time versions can maintain the performances of the original continuous time state observers. Indeed, several limitations associated with continuous time mode and the necessity of persistence excitations [1], [15]. These restrictions make some of the state observers not quite convenient for many domestic and industrial applications. For all early mentioned, those are considered motivations of sampled data observer design. Recently, sampled - data observer claimed by [12] is the central concern and it has an attractive research area for researchers and scientists in the last decade. For this reason, the present study will give some importance. In sampled - data design process, sampled - data are designed in the discrete time mode including zero order hold device (ZOH), whilst plant model designed in the continuous time mode. Latterly, inter sample behaviour coupled with high gain state observer to solve the problem of output measurements in continuous time mode during the design process. Output measurements are just accessible at sampling instant. Hence, to solve the problem of online state observation, this will lead us to combine the benefits of inter sampled behaviour with the high gain state observer. An approximate predictor for which the prediction is provided by the output of a system is claimed by [12]. Finally, the searcher will develop a new output feedback controller design dealing with sampled - data HGO running with variable speed induction drive to solve the problem of output measurements without resorting to use mechanical and magnetic sensors for online observation based on stator voltage and output currents. The stability convergence will be analyzed using the Lyapunov stability theory and input - to state stability concept. Compared to the classical high gain observer, the proposed observer involves a time varying gain. This gain plays a vital role in improving the dynamic performance of the proposed observer design. As reported in [9], a sensorless induction motor drives using sliding-mode state observer coupled with
output feedback controller had been implemented to observe the mechanical and magnetic state variables taking into consideration the continuous time measured stator currents. Nevertheless, there is chattering phenomenon in traditional sliding mode control. To solve this problem, many literates propose several different method for example a comparative study with conventional sliding mode control had been introduced to give scientific solutions for avoidance chattering phenomenon such as using saturation function instead of signum function and its application on induction machine. In fact, backstepping control design technique ensures the global stabilization of output feedback controller based on variable speed induction machine drive for whatever set of initial conditions.

The rest of the present paper is organized as: the nonlinear model of induction machine is given in section 2. Model transformation is provided in section 3. Whilst high gain observer design for a class of MIMO nonlinear system based induction machine is presented in section 4, the nonlinear controller design and the system analysis are presented in section 5, the dynamic behaviors are illustrated via simulation results in section 6.

2. MODEL OF INDUCTION MACHINE

The nonlinear mathematical model of sixth order variable speed three – phase induction machine projected on stationary (α–β) reference frame is given in the following dynamic state equations [4]:

\[
\begin{align*}
\frac{d\psi_{sa}}{dt} &= -\frac{R_s}{L_s} \psi_{sa} + \frac{N_s}{L_s} \psi_{ra} \psi_{ra} + \frac{1}{\sigma_s} u_{sa} \\
\frac{d\psi_{sb}}{dt} &= -\frac{R_s}{L_s} \psi_{sb} + \frac{N_s}{L_s} \psi_{rb} \psi_{rb} + \frac{1}{\sigma_s} u_{sb} \\
\frac{d\psi_{ra}}{dt} &= -\frac{R_s}{L_s} \psi_{ra} - \frac{1}{\sigma_s} \omega_m \psi_{rb} + n_p \omega_m \psi_{rb} + \frac{1}{\sigma_s} u_{sa} \\
\frac{d\psi_{rb}}{dt} &= -\frac{R_s}{L_s} \psi_{rb} - \frac{1}{\sigma_s} \omega_m \psi_{ra} + n_p \omega_m \psi_{ra} \\
\frac{d\omega_m}{dt} &= \frac{n_p M_{sr}}{J_m} \sigma \psi_{ra} \psi_{ra} - \frac{1}{J_m} \omega_m - \frac{1}{J_m} \omega_m \\
\frac{d\omega_l}{dt} &= \epsilon(t)
\end{align*}
\]

where, \( x \triangleq [i_{sa}, i_{sb}, \psi_{ra}, \psi_{rb}, \omega_m, T_r] \) denote the stator currents, rotor fluxes, motor speed, load torque vector.

\( u_s \triangleq [u_{sa}, u_{sb}]^T \) denote the stator voltage vector projected on stationary (α–β) reference frame. \( R_s, L_s, R_r, R_r \) are the stator and rotor electrical parameters. \( M_{sr} \) is the mutual inductance between the stator and rotor windings, \( n_p \) is the number of magnet pair poles, \( J_m \) is the moment of inertia, \( f_v \) is the viscous friction coefficient and \( \epsilon(t) \) is the unknown bounded function. The parameters \( \sigma, \gamma, \mu_r \) and \( N \) are defined as follows:

\[ \sigma \triangleq 1 - \frac{M_{sr}^2}{\sigma_s L_s^2}, \gamma \triangleq \frac{R_s}{\sigma_s L_s} + \frac{R_s M_{sr}^2}{\sigma_s L_s^2}, q_r \triangleq \frac{f_v}{R_s}, \text{and} \ N \triangleq \frac{M_{sr}}{\sigma_s L_s} \]

It is mentioned that the mathematical model in (1) is strongly nonlinear since it involves the product between rotor flux linkage and rotor speed. The major objective is to specify the conditions that the states of induction machine can be observed based on MIMO measurements, namely the stator voltage and sampled current measurements. This leads us to study the observability of the system given in (1) by letting, \( y \triangleq i_s \) as an output state vector. It gives the following:

\[
\begin{align*}
x &\triangleq [x_1, x_2, x_3]^T, \in \mathbb{R}^{3x2} \\
x_1 &\triangleq (x_{11}, x_{12})^T = (i_{sa}, i_{sb})^T, \in \mathbb{R}^2 \\
x_2 &\triangleq (x_{21}, x_{22})^T = (\psi_{ra}, \psi_{rb})^T, \in \mathbb{R}^2 \\
x_3 &\triangleq (x_{31}, x_{32})^T = (\omega_m, T_r)^T, \in \mathbb{R}^2
\end{align*}
\]

It should be confirmed that the notations:
- $I_k$ used to express ($k \times k$) identity matrix.
- $0_k$ the ($k \times k$) null matrix.
- $0_{k \times m}$ used to express ($k \times m$) null matrix.
Real system model in (1), disturbed by external load torque is:
\[
\begin{align*}
\dot{x} &= f(x, u_x) + B \varepsilon(t) \\
y &= Cx = x_1
\end{align*}
\]  
(4)  
where, $f(x, u_x)$ is the vector field function, $f \in \mathbb{R}^n \times \mathbb{R}^m$. It can be defined in the form:
\[
\begin{pmatrix}
    f_1(x, u_x) \\
f_2(x, u_x) \\
f_3(x, u_x)
\end{pmatrix} =
\begin{pmatrix}
    -\gamma x_1 + \frac{N}{\mu} g(x_{31}) x_2 + \frac{1}{\sigma \mu} u_x \\
    g(x_{31}) x_2 + \frac{M_r}{\mu} x_1 \\
m_{p\mu r} x_1^T J_2 x_2 - \frac{f_L}{I_m} x_{31} - \frac{1}{J_m} x_{32} - 0
\end{pmatrix}
\]  
(5)  
with, $B_1 = [0_{8 \times 1}]^T$, $C = [I_2 \ 0_2 \ 0_2], \forall \ I_2 \in \mathbb{R}^{2 \times 2}, y \in \mathbb{R}^{n_1}, J_2 \in \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$, and $g(x_{31}) = \frac{1}{\mu} I_2 - n_p \omega_m J_2$

3. MODEL TRANSFORM AND OBSERVABILITY TEST
It should define the sufficient conditions generated by observability criterion. Model transformation is formulated using the following transformation map:
\[
\Phi: \mathbb{R}^6 \rightarrow \mathbb{R}^6, x \rightarrow \zeta = \Phi(x) = [\Phi_1(x), \Phi_2(x), \Phi_3(x)]^T
\]  
(6)  
One can illustrate that the classical transformation in (6) puts system model in (4) under the canonical form:
\[
\begin{align*}
\dot{\zeta}_1 &= \zeta_2 + \varphi_1(\zeta_1, u_x) \\
\dot{\zeta}_2 &= \zeta_3 + \varphi_2(\zeta_1, \zeta_2) \\
\dot{\zeta}_3 &= \varphi_3(\zeta) \\
y &= C\zeta = \zeta_1(\text{output})
\end{align*}
\]  
(7)  
where, $\zeta = [\zeta_1 \ \zeta_2 \ \zeta_3]^T, \ z_k = [\zeta_{k1} \ \zeta_{k2}]^T$ with ($k = 1, 2, 3$). The nonlinear functions $\zeta_k$ and $\varphi_k \in \mathbb{R}^2$ are defined as:
\[
\begin{align*}
\zeta_1 &= \Phi_1(x) = x_1 \\
\zeta_2 &= \Phi_2(x) = N g(x_{31}) x_2 \\
\zeta_3 &= \Phi_3(x) = -n_p N J_2 (x_{31}) x_2 + \dot{x}_2 x_{13} \\
\end{align*}
\]  
(8)  
and the nonlinear functions are defined as:
\[
\begin{align*}
\varphi_1(\zeta_1, u_x) &= -\gamma \zeta_1 + \frac{1}{\sigma \mu} u_x \\
\varphi_2(\zeta_1, \zeta_2) &= -\zeta_2 + \frac{M_r}{\mu} \zeta_1 \\
\varphi_3(\zeta) &= \frac{\partial \Phi_3(x)}{\partial x_1} \dot{x}_1 + \frac{\partial \Phi_3(x)}{\partial x_2} \dot{x}_2 + \frac{\partial \Phi_3(x)}{\partial x_{31}} \dot{x}_{31} \\
b(\zeta) &= \frac{\partial \Phi_3(x)}{\partial x_{32}} \dot{x}_{32} = \frac{n_p N}{I_m} J_2 x_2
\end{align*}
\]  
(9)  
Now, let us consider $J_\Phi(x)$ be the Jacobian matrix of, $\Phi(x)$.

In view of (8), one has the following matrix for nonlinear systems with respect to closed-loop trajectories:
\[
J_\Phi(x) = \begin{pmatrix}
    I_2 & 0_2 & 0_2 \\
    0_2 & \frac{\partial \Phi_2(x)}{\partial x_1} & \frac{\partial \Phi_2(x)}{\partial x_2} \dot{x}_2 \\
    0_2 & \frac{\partial \Phi_2(x)}{\partial x_2} & \frac{\partial \Phi_2(x)}{\partial x_{32}} \dot{x}_{32}
\end{pmatrix} \in \mathbb{R}^{6\times6}
\]  
(10)  
Apparently that the Jacobian matrix has full rank, if and only if the following square matrix has full - rank:
\[
N_\Phi(x) = \begin{pmatrix}
    \frac{\partial \Phi_2(x)}{\partial x_1} & \frac{\partial \Phi_2(x)}{\partial x_2} \\
    \frac{\partial \Phi_2(x)}{\partial x_2} & \frac{\partial \Phi_2(x)}{\partial x_{32}} \dot{x}_{32}
\end{pmatrix} \approx \begin{pmatrix}
    N_1(x) & N_2(x) \\
    N_3(x) & N_4(x)
\end{pmatrix} \in \mathbb{R}^{4\times4}
\]  
(11)  
Once again, in view of equation (8), one has:
\[
\begin{align*}
\frac{\partial \Phi_2}{\partial x_2} &= N g(x_{31}) \\
\frac{\partial \Phi_2}{\partial x_{31}} &= \frac{\partial \Phi_2}{\partial x_{32}} \dot{x}_{32} = [0_{2 \times 1}] \\
\frac{\partial \Phi_3}{\partial x_1} &= \frac{M_p}{\mu} x_{31} + N \frac{n_p^2 M_r}{I_m L_r} x_2 \\
\frac{\partial \Phi_3}{\partial x_2} &= -n_p N J_2 \left( x_{31} \frac{\partial \Phi_3}{\partial x_1} \dot{x}_1 + \frac{n_p M_r}{I_m L_r} (x_2) J_2 x_2 + n_p N J_2 x_{31} g(x_{31}) \right) \\
\frac{\partial \Phi_3}{\partial x_{31}} &= -n_p N J_2 \left[ \dot{x}_2 + n_p x_{31} J_2 x_2, - \frac{1}{J_m} x_2 \right]
\end{align*}
\]  
The determinant of the matrix $N_\Phi(x)$ could be expressed as:
\[
det N_\Phi(x) = \det(N_1 N_4 - N_2 N_3)(x)
\]  
(12)  
Then, the considered state transformation has full rank if the following conditions are fulfilled as:
\[
\det(N_{\theta(s)}) = (-n_pN)^3 \dot{x}_{21} \left( \frac{1}{\mu_F^2} + (x_{31})^2 \right)
\]
\[
\text{or, } \det(N_{\theta(s)}) = (-n_pN)^3 \dot{x}_{22} \left( \frac{1}{\mu_F^2} + (x_{31})^2 \right)
\]

In case of constant motor speed (i.e. \( \dot{x}_{31} = 0 \)), a sufficient condition for local observability as the rotor fluxes:
\[
\dot{x}_{21} \neq 0 \text{ or } \dot{x}_{22} \neq 0
\]

One deduces that the system in (7) is uniformly observable system for any input. Thus, the system defined in (1) is observable in the rank sense if the state transformation in (6) exists [1].

4. HIGH GAIN OBSERVER FOR LM MACHINE

The major target of the current section is to propose an observer for nonlinear system stated in (1). Such observer used to provide online the demanding estimates of \( \omega_m \) and \( T_L \) including the rotor flux position based on measurements of the stator currents at the last sampling instant and stator voltages. If the online estimates of the rotor flux linkage \( (\psi_{ra}, \psi_{rB})^T \) become available, the mechanical rotor flux position could be estimated using the arctangent function without using small signal approximation:
\[
\theta_r \pm \frac{1}{n_p} \theta_e = \frac{1}{n_p} \arctan \left( \frac{\psi_{rB}}{\psi_{ra}} \right)
\]

where, \( \theta_r \) and \( \theta_e \) denote, respectively the mechanical rotor position and electrical position estimate measured in (rad).

A. Problem Formulation

The system model in (7) will express in the following structure for Lipschitz systems. It consists of linear and nonlinear terms.

\[
\begin{align*}
\dot{\zeta} &= A\zeta + \varphi(\zeta, u_s) \\
y &= C\zeta = \zeta_1 \text{ (output)}
\end{align*}
\]

where, the state variable, \( \zeta \triangleq (\zeta_1 \xi_2) \) is defined as anti - shift block diagonal matrix, where \( 0_2 \) and \( I_2 \) are \( (2 \times 2) \) zero and identity matrix, respectively:
\[
A = \begin{bmatrix}
0_2 & I_2 & 0_2 \\
0_2 & 0_2 & I_2 \\
0_2 & 0_2 & 0_2
\end{bmatrix} \in \mathbb{R}^{6 \times 6}
\]

The general structure of the high gain observer having time varying gain and discrete time correction term is [1].

\[
\begin{align*}
\dot{\xi} &= A \xi + \varphi_d(\xi, u_s) - \theta \Delta^{-1}K\varepsilon(t) \left( C\xi(kt) - y(kt) \right) \\
\dot{\varepsilon} &= -\xi\varepsilon(t) ^{\text{b}} \\
\varepsilon(t_k) &= 1, \quad \forall t \in [kt, (k + 1)t), k \in \mathbb{N}
\end{align*}
\]

where, \( \theta > 1 \) is a high gain observer design parameter and \( \xi, \text{ b} \) are scalar constants \( \xi > 0, \quad 0 < b < 1 \). The sampled vector is accessible at successive sampling instant.

Before giving the state equations of the proposed high gain observer, one introduces some important definitions:

- The block diagonal matrix, \( \Delta \) is defined by the following form:
\[ \Delta \triangleq \text{blockdiag}(\mathbb{I}_2, \frac{1}{\sqrt{2}} \mathbb{I}_2, \frac{1}{\sqrt{2}} \mathbb{I}_2) \] 

with, \( \theta > 0 \), is a constant HGO parameter and \( \mathbb{I}_2 \in \mathbb{R}^{2 \times 2} \).

The fixed gain matrix \( K \in \mathbb{R}^{6 \times 2} \) can be chosen:

\[ K^T \triangleq [k_1 \mathbb{I}_2, \ k_2 \mathbb{I}_2, \ k_3 \mathbb{I}_2] \]

with, \( k_i \in \{1, 2, 3\} \) is a real positive scalar so that \( (A - KC) \) is Hurwitz and it satisfies the following algebraic Lyapunov equation for solving symmetric positive definite matrix:

\[ P(\Delta K) + (A - KC)^T P = -Q = \mu \mathbb{I}_2, \ \mu > 0 \]

where, \( (P, Q) \in \mathbb{R}^{6 \times 6} \times \mathbb{R}^{6 \times 6} \) is a pair of symmetric positive definite matrices and \( (A - KC)^T \) is the transposed matrix for \( P(\Delta K) \).

The vector \( \xi \) is the estimate of the real state.

**B. Stability Analysis of HGO**

**Theorem 1 (Main result):** Consider the class of system (4) related to three – phase induction machine drive and suppose that \( A_1, A_2 \) hold. There exist two positive constant design parameters, \( \theta_0 \) and \( \tau \) so that \( \forall \theta > Sup \{1, \theta_0\}, \forall 0 < \tau \leq \tau_{\text{max}} \). Thus, the system in (16) is globally exponentially observer of system (15). Subsequently, the observation error tends to origin exponentially for any initial conditions.

**Proof:** For writing convenience, the time index will be cancelled. The proposed observer (16) is rewritten in terms of the following observation error \( \hat{\xi} \triangleq \xi - \hat{\xi} \) as:

\[
\begin{align*}
\dot{\xi} &= \theta A\xi + \Delta \left( \phi_\circ(\xi, u) - \phi(\xi, u) \right) - \theta \int_{\phi(x)}^\Delta A^{-1} K e(t) \varphi(kr) \\
\dot{\hat{\xi}} &= -\xi c(t)^b \quad \forall \ t \in [kr, (k + 1)\tau), \ k \in \mathbb{N} \\
c(t_k) &= 1
\end{align*}
\]

Let us introduce the change of coordinates, \( \xi \triangleq \Delta \xi \) and the mathematical identities:

\[ \Delta A^{-1} = \theta A, \ \text{and} \ C\Delta = \Delta^{-1} = C. \]

The system in (20) can be rewritten in terms of \( \hat{\xi} \) as in the following form:

\[
\begin{align*}
\dot{\hat{\xi}} &= \theta A\hat{\xi} + \Delta \left( \phi_\circ(\hat{\xi}, u) - \phi(\hat{\xi}, u) \right) - \theta \int_{\phi(x)}^\Delta A^{-1} K e(t) \varphi(kr) \\
\dot{c}(t) &= -\xi c(t)^b \quad \forall \ t \in [kr, (k + 1)\tau), \ k \in \mathbb{N} \\
c(t_k) &= 1
\end{align*}
\]

Now, let the measurement error, \( z(t) \triangleq \bar{y}(t) - c(t)\varphi(kr) \). It is apparent at the sampling instant. \( z(k\tau) = 0 \). One obtains the augmented error system dynamics:

\[
\begin{align*}
\dot{\hat{\xi}} &= \theta A\hat{\xi} + \Delta \left( \phi_\circ(\hat{\xi}, u) - \phi(\hat{\xi}, u) \right) - \theta \int_{\phi(x)}^\Delta A^{-1} K e(t) \varphi(kr) \\
\dot{z}(t) &= \theta \xi + \varphi_\circ(\hat{\xi}, u) - \varphi_\circ(\xi, u) - \theta \int_{\phi(x)}^\Delta \varphi_\circ(\xi, u) \varphi(kr) - \varphi_\circ(t, \varphi(kr)) \\
\dot{c}(t) &= -\xi c(t)^b \quad \forall \ t \in [kr, (k + 1)\tau), \ k \in \mathbb{N} \\
c(t_k) &= 1
\end{align*}
\]

Now, this system is analyzed using the Lyapunov function candidate, \( V \triangleq \xi^T P \xi \). It is checked via algebraic Lyapunov equation in (19), the dynamics of Lyapunov function along the trajectories of (22), it gives:

\[
V \leq -\mu \theta \|\xi\|^2 + 2\sqrt{3} P \Delta L \|\xi\|^2 + 2\theta^T PKz(t)
\]

Using \( A_1 \), one gets for third order system model an inequality:

\[
\dot{V} \leq -\mu \theta \|\xi\|^2 + 2\sqrt{3} P \Delta L \|\xi\|^2 + 2\theta^T PKz(t)
\]

Let, \( w \triangleq \sqrt{V} \), the above inequality has:

\[
\dot{W} = 0.5 \cdot V(V)^{-0.5} \\
\dot{W} \leq -\mu \theta \|\xi\|^2 + 2\sqrt{3} P \Delta L \|\xi\|^2 + 2\theta^T PKz(t)
\]

Using the fact, \( \lambda_{km} \|\xi\|^2 \leq V \leq \lambda_{pm} \|\xi\|^2 \), one gets:

\[
\dot{W} \leq -\mu \theta \|\xi\|^2 + 2\sqrt{3} P \Delta L \|\xi\|^2 + 2\theta^T PKz(t)
\]
Choosing the observer design parameter, 
\[ \frac{\mu_0}{4} > \frac{\lambda \rho_m}{\lambda ho_m} \sqrt{3} |\beta_1| |P|, \]
Or equivalently, \( \theta > \theta_0 = \sup \left\{ \frac{1}{\lambda \rho_m} \left( \frac{\sqrt{3} |\beta_1| |P|}{\lambda \rho_m} \right) \right\} \).
One deduces, \( \forall \theta > \theta_0 \):
\[ W \leq -\frac{\mu_0}{\lambda \rho_m} W + \frac{\mu_0 |\beta_1| |P|}{\lambda \rho_m} \]
(27)
Introducing the mathematical notation, 
\[ \rho_0 \triangleq \frac{\mu_0}{\lambda \rho_m} \; \text{and} \; \rho_1 \triangleq \frac{\mu_0 |\beta_1| |P|}{\lambda \rho_m} \]
Now, in view of (22), there are two subsystem dynamics related to state observation error and measurement error at the sampling instant \( t \in [k \tau, (k + 1) \tau) \). They will be analyzed in two steps with the assistance of small gain condition [2].

**Step 1:** Proof the mapping \( z \mapsto \tilde{z} \) is input to state stable (ISS).
Integrating both sides of (27), it gives the general solution:
\[ W(t) e^{-\rho_0 \int_{t_0}^{t} e^{-\rho_0 (t-s)} ds} = W(t_0) e^{-\rho_0 t_0} + \int_{t_0}^{t} e^{-\rho_0 (t-s)} |z(s)| ds \]
(28)
Letting, \( 0 < \rho < \frac{\rho_0}{2} \); one writes:
\[ W(t) e^{-\rho \rho_0 t} W(t_0) e^{-\rho_0 t_0} \leq \frac{2 \rho_1}{\rho_0} \sup_{t_0} \sup_{t_0} (e^{-\rho (t-s)} |z(s)|) \]
(29)
Let us consider, \( M(t_0) \triangleq W(t_0) e^{-\rho_0 t_0} \), it gives:
\[ ||\tilde{z}(t)|| \leq e^{-\rho t} M(t_0) + \frac{2 \rho_1}{\rho_0 \sqrt{\lambda \rho_m}} \sup_{t_0} (e^{-\rho (t-s)} |z(s)|) \]
One can limit the left-hand side of (30) as:
\[ \sup_{t_0} (e^{-\rho t} ||z(t)||) \leq \frac{M(t_0)}{\sqrt{\lambda \rho_m}} + \frac{2 \rho_1}{\rho_0 \sqrt{\lambda \rho_m}} \sup_{t_0} (e^{-\rho (t-s)} |z(s)|) \]
(30)
From (30), the mapping \( z \mapsto \tilde{z} \) is input to state stable ISS.

**Step 2:** Proof the mapping \( \tilde{z} \mapsto z \) is (ISS).
One first integrates the following equation given in (22), as follows:
\[ z(t) = \int_{t_0}^{t} \left( \theta \tilde{z}(s) + \varphi(s) \xi(s, u) - \varphi_1(\xi(s), u) \right) ds \]
\[ - \int_{t_0}^{t} \left( \theta C \xi_0(s) \xi_0(kr_0) - \xi_0(s) \xi_0(kr_0) \right) ds + \int_{t_0}^{t} \left( \theta \xi_0(s) + \xi_0(s) \right) ds \]
(31)
Note that \( |c(s)| \leq 1, |c| = 1 \). Using, \( A_1 \), one has:
\[ e^{\alpha t} |z(t)| \leq te^{\alpha t} |\theta + L + \theta |K| + |z| \sup_{t_0} (e^{\alpha t} ||z(t)||) \]
\[ Or, \sup_{t_0} (e^{\alpha t} |z(s)|) \leq te^{\alpha t} |\theta + L + \theta |K| + |z| \sup_{t_0} (e^{\alpha t} ||z(t)||) \]
(32)
In view of equation (32), \( t \geq t_0, \forall i = 0, \ldots, j \).
It means that:
\[ \sup_{t_0} (e^{\alpha t} |z(s)|) \leq te^{\alpha t} |\theta + L + \theta |K| + |z| \sup_{t_0} (e^{\alpha t} ||z(t)||) \]
(33)
It is obvious from (33), the mapping \( \tilde{z} \mapsto z \) is ISS. Now, combining the result of step 2 with the last result of step 1, it gives the observation error:
\[ \sup_{t_0} (e^{\alpha t} |z(s)|) \leq e^{-\rho t} \frac{M(t_0)}{\lambda \rho_m} + \frac{2 \rho_1}{\rho_0 \sqrt{\lambda \rho_m}} \]
(34)
One can write (34) as follows:
\[ \sup_{t_0} (e^{\alpha t} ||z(t)||) \leq \frac{M(t_0)}{\sqrt{\rho_0 \lambda \rho_m}} e^{-\rho t} \]
(36)
To ensure validation of (36) and using small gain condition, it gives:
\[ re^{\alpha t} |\theta + L + \theta |K| + |z| \frac{2 \rho_1}{\rho_0 \sqrt{\lambda \rho_m}} \leq 1 \]
(37)
Finally, \( ||z(t)|| \leq \Delta^{-1} \frac{M(t_0)}{\sqrt{\rho_0 \lambda \rho_m}} e^{-\rho t} \]
(38)

5. BACKSTEPPING CONTROLLER DESIGN
Backstepping design technique generates a corresponding error variable, which can be stabilized by proper selection input via Lyapunov theory.
In fact, one has two control objectives with two control inputs named by, \( u_1 \) and \( u_2 \) for rotor speed and flux regulator [1].

**Step 1:** The following speed tracking error is:
\[ \hat{\xi}_1 \triangleq \omega_m^* - \hat{\omega}_m, \; \hat{\xi}_2 \triangleq \psi^*_2 - \hat{\psi}_2 \]
(39)
In view of system model defined in (1), the above error submits to:
Obviously that the actual control laws are expressed in system (47). Hence, the augmented Lyapunov function is:

$$W_2 \pm 0.5 * (\hat{\epsilon}_1^2 + \hat{\epsilon}_2^2 + \hat{\epsilon}_3^2 + \hat{\epsilon}_4^2)$$ (48)

The time derivative of the augmented Lyapunov function is:

$$\dot{W}_2 = -h_1 \hat{\epsilon}_1^2 - h_2 \hat{\epsilon}_2^2 - h_3 \hat{\epsilon}_3^2 - h_4 \hat{\epsilon}_4^2$$ (49)

with $h_3$ and $h_4$ are positive design parameters used to guarantee negative definiteness of second Lyapunov function. Thus:

$$\dot{W}_2 = -\epsilon_1^2 - \epsilon_2^2 - \epsilon_3^2 - \epsilon_4^2 \leq 0$$ (50)

The real control laws that ensure globally exponentially stable of the system (50) are:

$$u_{sa} = \frac{1}{\psi_f^2} \left[ \frac{(M_4 + \hat{\epsilon}_4 + h_4 \hat{\epsilon}_4)}{2NR_r} \psi_{ra} - \frac{J_m}{n_p N_1} (M_3 + \hat{\epsilon}_2 + h_3 \hat{\epsilon}_4) \psi_{rb} \right]$$

$$u_{sb} = \frac{1}{\psi_f^2} \left[ \frac{(M_4 + \hat{\epsilon}_4 + h_4 \hat{\epsilon}_4)}{2NR_r} \psi_{rb} + \frac{J_m}{n_p N_1} (M_3 + \hat{\epsilon}_2 + h_3 \hat{\epsilon}_4) \psi_{ra} \right]$$ (51)

Let us define, $u_i \equiv \overline{u_{sa}}$ and $u_{2} = \overline{u_{sb}}$, which represent the averaged model in ($\alpha$ – $\beta$) axis of the duty ratio system $(S_1, S_2, S_3)$ [15]:

$$S_i \equiv \begin{cases} 1 \text{ if } S_i \text{ ON and } \overline{S_i} \text{ OFF } & i = 1, 2 \end{cases}$$ (52)

6. SIMULATION RESULTS AND VERIFICATION

A simulation experiments are implemented for three – phase induction motor drive to ensure validation of the candidate output feedback controller as described in Fig.1. The nominal values and characteristics are listed in Table I. The candidate high gain observer used for online estimation of unmeasured mechanical and magnetic state variables, respectively. The control design parameters [41, 44, 99, 100] and observer design parameter, $\theta = 90$ have been optimized using MATLAB environments as listed in
Table II. Hence, it is useful for providing the information about the rotor position for control system before and during the starting-up system model. The resulting dynamic behaviors of one operating cycle of 16s are shown in Figs. 2, 3, 4, 5 and 6 at sampling time interval of 4 ms. It is also noteworthy that they indicate the waveforms of tracking performance for the candidate observer are quite acceptable for sensorless variable speed induction machine drive. Fig. 2 associated with measured and estimated of stator currents in \((\alpha - \beta)\) coordinates. To get better imagination and performance of the proposed output feedback controller, it is observed that as the online estimates of the rotor flux linkage projected on \((\alpha - \beta)\) coordinates are available as shown in Fig. 3, the rotor flux position could be estimated after 0.1s as illustrated in Fig. 4. However, the measured, reference and estimated values of rotor speed converge to the desired reference level with a good accuracy as shown in Fig. 5 in response to the external load torque variation between [15, 21] N.m as shown in Fig. 6. The rotor speed estimate follows its desired reference signal with acceptable accuracy. The output feedback control for first and second control inputs are simulated as shown in Fig. 7, which is bounded to ensure boundedness of physical variables. They are affected by measurement noise with sufficiently large spectrum of white noise with 7 mV peak value for stator voltages. Fig. 8 shows the dynamic performances of time varying gain for three different cases, \(i.e. b = [1,0.75,0.25]\). It is observed that the sampling time interval increases, \(i.e. \tau = [2,2.8,4]\), respectively with decreasing the parameter, \(b\) in which the time varying gain, \(c(t)\) becomes faster resetting as listed in Table II. The origin of the closed loop system under output feedback controller is globally exponentially stable as operating time closes to infinity for random set of initial conditions.

### TABLE I. INDUCTION MACHINE CHARACTERISTICS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated power</td>
<td>(P_r)</td>
<td>7.5 kW</td>
</tr>
<tr>
<td>Nominal flux</td>
<td>(\psi_n)</td>
<td>1 Wb</td>
</tr>
<tr>
<td>Stator resistance</td>
<td>(R_s)</td>
<td>0.63 (\Omega)</td>
</tr>
<tr>
<td>Rotor resistance</td>
<td>(R_r)</td>
<td>0.4 (\Omega)</td>
</tr>
<tr>
<td>Stator inductance</td>
<td>(L_s)</td>
<td>0.097 H</td>
</tr>
<tr>
<td>Rotor inductance</td>
<td>(L_r)</td>
<td>0.091 H</td>
</tr>
<tr>
<td>Mutual inductance</td>
<td>(M_{sr})</td>
<td>0.091 H</td>
</tr>
<tr>
<td>Viscous friction</td>
<td>(f_v)</td>
<td>0.001 N.s / rad</td>
</tr>
<tr>
<td>Moment of inertia</td>
<td>(J_m)</td>
<td>0.022 kg. m(^2)</td>
</tr>
<tr>
<td>Pole pairs</td>
<td>(n_p)</td>
<td>2</td>
</tr>
</tbody>
</table>

### TABLE II. PARAMETERS OF THE PROPOSED OBSERVER AND CONTROLLER

<table>
<thead>
<tr>
<th>Parameter (K)</th>
<th>(\theta)</th>
<th>(\tau) (ms)</th>
<th>(b)</th>
<th>(\xi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value ([20 65 85]^{T})</td>
<td>(\theta)</td>
<td>2.5</td>
<td>1</td>
<td>0.75</td>
</tr>
<tr>
<td>Parameters (h_1)</td>
<td>(h_2)</td>
<td>(h_3)</td>
<td>(h_4)</td>
<td></td>
</tr>
<tr>
<td>Value</td>
<td>41</td>
<td>44</td>
<td>99</td>
<td>100</td>
</tr>
</tbody>
</table>
Fig. 1. Induction motor drive running with high gain state observer

Fig. 2. Measured, and estimated \((\alpha - \beta)\) currents measured in (A)

Fig. 3. Measured and estimated \((\alpha - \beta)\) rotor flux measured in (Wb)

Fig. 4. Estimated and real rotor flux position measured in (rad)

Fig. 5. Reference, measured and estimated rotor speed measured in (rad/sec)

Fig. 6. Measured and estimated load torque variation measured in (N. m)
CONCLUSIONS

Design of sampled – data observer, based on the machine nonlinear model and backed by a formal analysis, has yet to be solved. A solution to this problem is developed by combining ideas from the high gain observer and input to - state stability technique. It is highly recommended to design an observer for online estimation of non-measurable mechanical and magnetic state variables of the AC machine. Thus, one must exploit the benefits and technical features of high gain observer associated with fast exponential convergence, accurate estimates and sampled vector.

To ensure validation the main results described by theorem (main result), a nonlinear output feedback controller of the induction machine drive has been address in $(\alpha – \beta)$ coordinates. The novelty of this work resides in the proposed state observer includes a resetting adaptation gain that will reflect positively on increasing sampling time interval. These results can be applied to an adaptive high gain state observer and other classes of uniformly observable nonlinear systems.

Based on simulation results, a considerable transient oscillation is seen in estimated mechanical, magnetic and some of electrical state variables under rapid acceleration and unknown load disturbance. By adding the arctangent computation to get rotor flux position error, this transient oscillation will be omitted with time progressive, consequently to improved sensorless induction machine drive dynamic behaviors.

REFERENCES


gain for a class of nonlinear systems with sampled measurements,” IEEE conference on decision and control, pp. 316 – 321.


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observers for electrical systems taking into
account the time varying gain based on tools of
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mechanical and magnetic sensors and their
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electric drives, and smart grids. He seeks for
developing new concepts in smart technologies.

تصميم وحدة تحكم تغذية راجعة لسواقة محركات كهربائية حثية متغيرة السرعة
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الخلاصة:
أن الغرض الرئيسي من هذه الدراسة هو تصميم وحدة تحكم تغذية راجعة استنادا إلى عينات - إخراج كاملاً للرؤية
لمرافق حالة نوع عالي الربح متغيع مع محرك حثي متغير السرعة ذو موديل رياضي لاختي. مرافق الحالة الجديد
المقترح ضمن المعيار القياسي المتداول عالميا يكون قوي نسبة إلى وقت أخذ العينات وعزم الدوران الخارجي الغير
معروف إلى مرافق الحالة. فقط اثنين من المتغيرات الكهربائية من المتغيرات المعروفة أن تكون في معلومة وهي فولتية الجزء
الثابت وتيارات الجزء الثابت في الوقت المستقبلي. تقدم هذه الدراسة تحليل الملاحظات لمحرك حثي ثلاثي الطور مما
يؤدي إلى أعداد أشرطة كافية ومقبول لخصائص القدرة المتزامنة. تضمن تقنية التحكم الاخطية سرعة تتبع وقت الاستجابة
والتقريب الامتصال الإساسي من خطأ المراقبة. تم تجربة تقريب الاستقرارية لمرافق الحالة المفترض باستخدام أساليب
نظرية الاستقرار المصممة. أثبتت تجربة الملاحظة باستخدام مبدأ مثال دعم النتائج الرئيسية وضمان كفاءة
الاستراتيجية المفترضة على مواسفة محاكاة منصب لو سعة 7,5 كيلو واط مستندًا على تصميم مراقب حالة بحفل
ميززة الربح المتغيرة مع الزمن. تعلم هذه الميزة دور فاعلا في زيادة وقت أخذ العينات مقارنة مع خاصية الربح الثابت
مع الزمن مما إدى إلى زيادة زمن العينة أبداء من 1.5 ms إلى 2.5 ms.

الكلمات الرئيسية: مراقب حالة محرك حثي, نظرية ليابونوف, عينات - إخراج, وحدة تحكم تغذية راجعة.

List of Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>state transition matrix</td>
</tr>
<tr>
<td>B</td>
<td>gain matrix corresponding to input control signals</td>
</tr>
<tr>
<td>Ce</td>
<td>correction error</td>
</tr>
<tr>
<td>C</td>
<td>output state vector</td>
</tr>
<tr>
<td>f</td>
<td>main supply frequency in Hz</td>
</tr>
<tr>
<td>f(·)</td>
<td>vector field function</td>
</tr>
<tr>
<td>f_r</td>
<td>combined rotor and load viscous friction</td>
</tr>
<tr>
<td>h_i</td>
<td>constant design parameter, , i = 1:2</td>
</tr>
</tbody>
</table>
\( l_{s\alpha}, l_{s\beta} \)  
\( \mathbb{I}_n \)  
\( J_{\Phi(x)} \)  
\( J \)  
\( K \)  
\( n_p \)  
\( \mathbb{R} \)  
\( V_{sat}, V_{sp} \)  
\( \zeta \)  
\( \hat{\zeta} \)  
\( \tilde{\zeta} \)

Stator currents projected on \((\alpha - \beta)\) representation in A
Identity matrix size \((n \times n)\)
Jacobian matrix of the PMSM system
Combined rotor and load moment of inertia
Gain matrix corresponding to measurements error
Sampling sequence
Number of magnet pole – pairs
Set of real numbers
\((\alpha - \beta)\) axis stator voltage components in V
Real state vector
State estimate
State observation error